B.Sc. (CBCS Pattern) Semester-V

USMT09 DSE- Mathematics Paper-I - Linear Algebra

P. Pages : 2 **GUG/S/25/13115**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that intersection of two subspaces of vector space is a subspace. **6**

b) Let A and B be two nonempty subsets of vector space V then prove that **6**

i) $L(A \cap B) \subseteq L(A) \cap L(B)$

ii) $L(A \cup B) = L(A) + L(B)$

OR

c) If $v_1, v_2, ...v_n$ is a basis of vector space V or span V over field F and if $w_1, w_2, ...w_m \in V$ are L. I. then prove that $m \le n$. **6**

d) If U and W are finite dimensional subspace of vector space V then prove that **6**

i) U + W is finite dimensional.

ii) dim (U+W) = dim U + dimW - dim $(U \cap W)$

UNIT – II

2. a) Let U and V be the vector spaces over field F and $T : U \rightarrow V$ be linear map then prove that **6**

i) T(0) = 0

ii) T(-u) = - T(u)

iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + ...\alpha_n u_n)$
$= \alpha_1 T(u_1) + \alpha_2 T(u_2) + ... + \alpha_n T(u_n)$
$\forall u_i \in u, < i \in F, 1 \le i \le n$ and $n \in N$

b) If $T : v_2 \rightarrow v_4$ be linear map defined by T(1, 1) = (0, 1, 0, 0) and T(1, -1) = (1, 0, 0, 0) where {(1, 1), (1, -1)} is a basis of v_2 find T(x, y) **6**

OR

c) Let $T : U \rightarrow V$ be a linear map then prove that: **6**

i) R(T) is a subspace of V

ii) N(T) is a subspace of U

d) Let $T : v_3 \rightarrow v_3$ be a linear map defined by $T(\rho_1) = \rho_3, T(\rho_2) = \rho_1, T(\rho_3) = \rho_2$ where ρ_1, ρ_2, ρ_3 are standard basis of v_3 then prove that $T^2 = T^{-1}$. **6**

UNIT – III

3. a) Let V be the finite dimensional vector space over F then prove that $V \approx \hat{\hat{V}}$ **6**

b) Let W be a subspace of a finite dimensional vector space V then prove that A(A(w)) = w **6**

OR

c) If w_1 and w_2 are subspaces of a finite dimensional vector space V then show that $A(w_1 + w_2) = A(w_1) \bigcap A(w_2)$ **6**

d) Let V be a vector space over F. For a subset S of V, Let **6**

$A(s) = \{f \in \hat{V} / f(s) = 0, \ \forall s \in s\}$ Prove that A(s) = A(L (s)) where L(s) is linear span of S

UNIT – IV

4. a) In $F^{(n)}$ define, for $u = (\alpha_1, \alpha_2 .... \alpha_n)$ and **6**

$v = (\beta_1, \beta_2, ... \beta_n)$,

$(u, v) = \alpha_1 \overline{\beta}_1 + \alpha_2 \overline{\beta}_2 + ... + \alpha_n \overline{\beta}_n$

Show that this defines on inner product.

b) Let V be an inner product space over F If $u, v \in V$ then prove that $|(u, v)| \le \| u \| \cdot \| v \|$. **6**

OR

c) Let W be a subspace of vector space V over field F then prove that $w^{\perp}$ is a subspace of V. **6**

d) By using Gram-Schmidt orthogonalization process, orthonormalize the L. I. subset $\{(1,1,1), (0,1,1), (0,0,1)\}$ of v_3 **6**

5. Solve **any six**.

a) Let V be a vector space over F then prove that $(-\alpha) \vee = -(\alpha v)$ for $\alpha \in F, \vee \in V$ **2**

b) In V_2 show that (3, 7) $\in$ [(1,2), (0, 1)] **2**

c) Let $T : U \to V$ be a linear map, Then prove that T is one-one $\Leftrightarrow$ $N(T) = \{0\}$ **2**

d) State Rank-Nullity theorem in a vector space. **2**

e) Prove that A(w) is a subspace of $\hat{V}$ **2**

f) If U and w be subspace of V over F then prove that $U \subset w \Rightarrow A(w) \subset A(U)$ **2**

g) Prove that $(u, \alpha v + \beta w) = \overline{\alpha} \ (u, v) + \overline{\beta}(u, w) \ \alpha_1 \beta \in F \ u_1 v \in V$ **2**

h) Prove that, for subspace w in a vector space, $w \bigcap w^{\perp} = \{0\}$ **2**

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